Addition and subtraction

Basic to subject specific (Beck's Tiers):

+, add, addition, more, plus make, sum, total altogether, double, near double, one more, two more... ten more... one hundred more, =, equals to, sign, is the same as, Tens, ones, partition, near multiple of 10, tens boundary, more than, less than, fewer, difference, one more, two more… ten more… one hundred more
Subtraction, subtract, take away, difference, difference between, minus, fewer, Tens, ones, partition , Less than, one less, two less… ten less… one hundred less
More, one more, two more... ten more... one hundred more

Instructional vocabulary:
tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of… show how you…

Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd + odd = even; odd + even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.

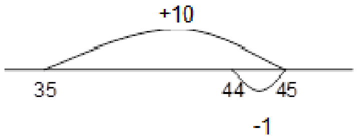
Some Key Questions: What's the same? What's difference? What do you notice? What patterns can you see?
How many altogether? How many more to make…? How many more is… than…? How much more is…?
Is this true or false?
If I know that 17 + 2 = 19, what else do I know? (e.g. 2 + 17 = 19; 19 – 17 = 2; 19 – 2 = 17; 190 – 20 = 170 etc).

NC 2014: Solve problems with addition and subtraction:
Applying their increasing knowledge of mental and written methods.
Recall and use addition and subtraction facts to 20 and 100:
Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two- digit number and tens;Two, two-digit numbers including regrouping;
Adding three one-digit numbers.
Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot.
Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

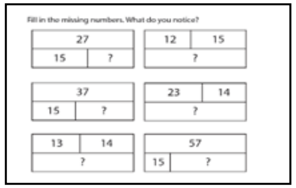
Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting forwards in tens from any number should lead to adding multiples of 10.

Number lines should continue to be an important image to support mathematical thinking, for example to model how to add 9 by adding 10 and adjusting.

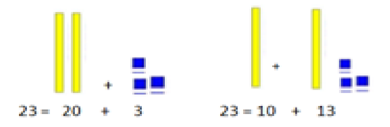


Children should practise addition to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g using 7 + 3 = 10 to find 17 + 3= 20, 70 + 30 = 100
They should use concrete objects such as bead strings and number lines to explore missing numbers 45 + = 50.



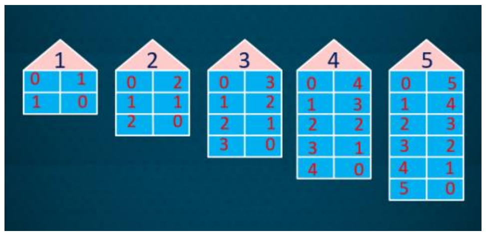
As well as number lines, 100 squares could be used to explore patterns in calculations such as 74 +11, 77 + 9 encouraging children to think about ‘What do you notice?’ where partitioning or adjusting is used.

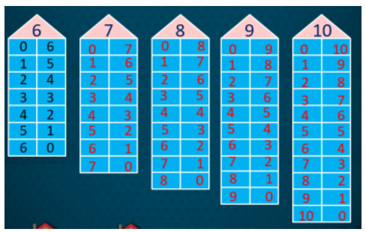
Children should learn to check their calculations, by using the inverse.
They should continue to see addition as both combining groups and counting on.
They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g. 23 = 20 + 3 = 10 + 13. Children should use jottings to show their understanding.

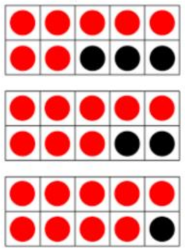


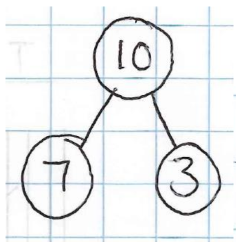
Number bonds within ten and to ten

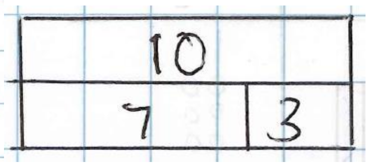
Practise regularly and show all facts using tens frame. Use the cherry representation and the bar model to expose the structure further. Record the addition and subtraction number sentences.

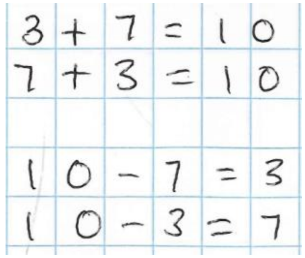






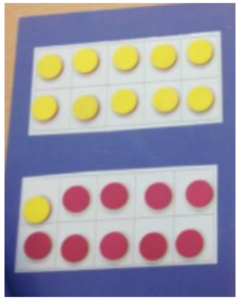


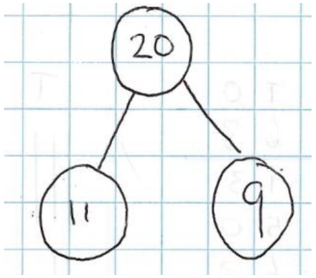


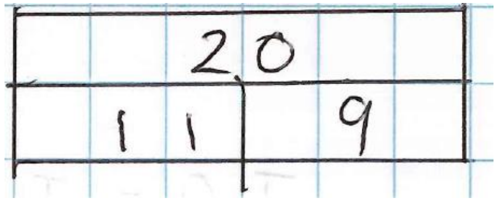


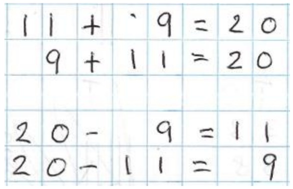
Addition and subtraction facts of numbers up to 20.

Practice regular and show all facts using tens frame. Use the cherry representation and the bar model to expose the structure further. Record the addition and subtraction number sentences.





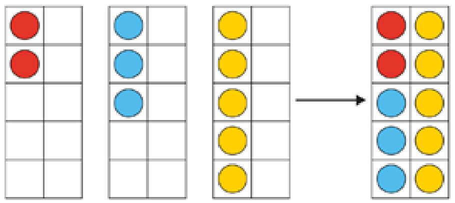




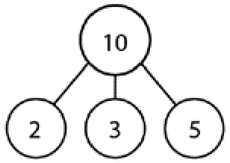
Addition of three addends using aggregation – combining of two or more quantities (How much/many altogether? What is the total?).

'Madison has two red marbles, Charlie has three blue marbles and Asif has five yellow marbles. They have ten marbles altogether.'

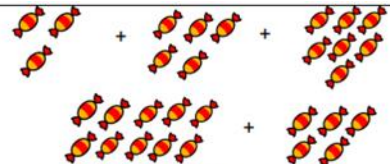
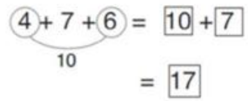
Tens frames:



Part–part–part–whole representation:



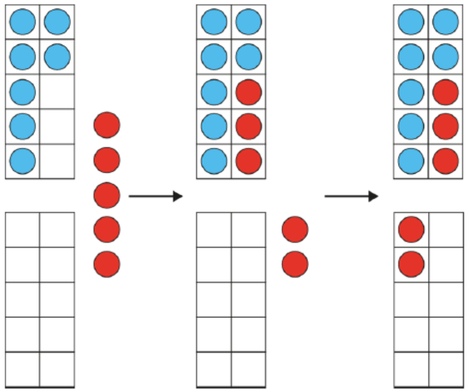
The ten frames exposes the visual representation of combing numbers. The part-whole diagram should be used alongside to support abstract notation.

Adding three single digits	**4 + 7 + 6= 17** Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.

Adding 2 numbers to bridge ten (making ten strategy)

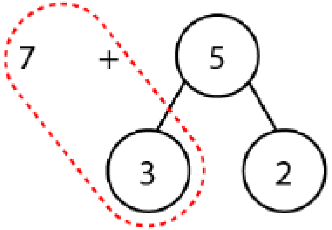
Introduce the idea that we can transform a sum of the two addends 7 + 5 into a sum of three addends 7 + 3 + 2, by partitioning one of the addends. Children need to be able to explain the steps using the part-part-whole cherry representation. Eg first I partition the five: three plus two is equal to 5. The seven plus three is equal to ten ect.

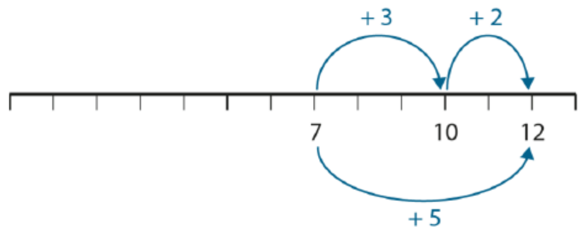
Tens frames:



$7 + 3 = 10$

$10 + 2 = 12$

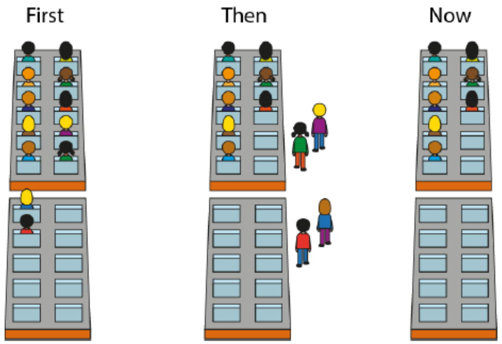




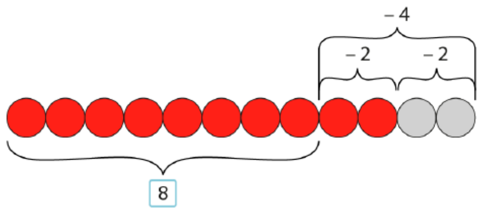
Also represent the strategy using a number line.

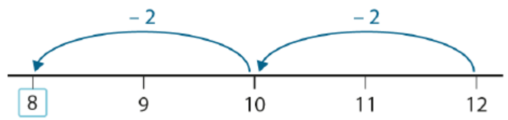
<u>Subtraction across tens boundary by subtracting *through* ten or subtracting *from* ten.</u>

Subtraction through ten, we subtract the down to ten, and then subtract the rest of the subtrahend from ten, for example 12 – 4 = 12 – 2 – 2 = 8



Subtraction *through* ten – bead bar and number line:





Model the process on a bead bar and number line, continuing to describe in full sentences. Make links between the tens frame and bead bar/ number line.

Children should have the opportunity to work through these stories/calculations using tens frame and verbalise the process using mathematical language eg ‘first partition the four into two and a two. Then subtract two from twelve to get to ten. Then subtract the remaining two from the ten.

Subtraction *through* ten – abstract:

12 – 4

2 2

$12 - 2 = 10$

$10 - 2 = 8$

so

$12 - 4 = 8$

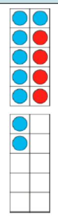
Model writing equations to express the two-stage process. Children develop their ability to jot down partitioning of the subtrahend and express the two steps.

Support children to develop this using examples below.

16 - 9 =

<u>Subtraction from ten</u>

Subtraction *from* ten:



12 – 4

10 2

$10 - 4 = 6$

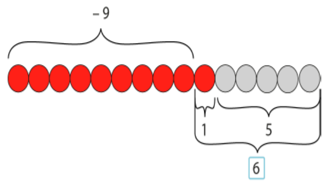
$6 + 2 = 8$

so

$12 - 4 = 8$

Subtraction *from* ten concept – bead bar representations:

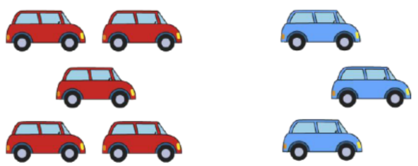
$15 - 9 = 6$



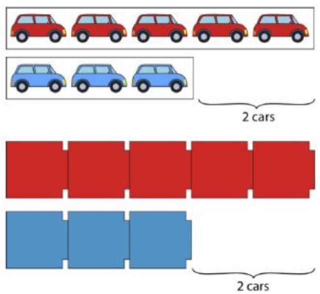
<u>Subtraction as difference:</u>

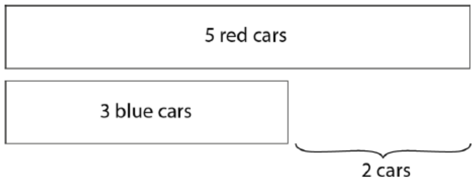
Using comparison to expose difference. Use a variety of examples to draw out a range of vocabulary for example, bigger, smaller, older, younger, more, less, fewer, heavier, lighter. Also remember to use ‘less’ for continuous variables and ‘fewer’ for discrete variables.

Discrete objects:



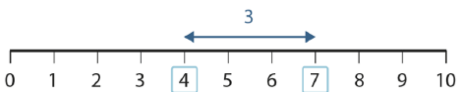
- *‘There are* more red cars than blue cars.’
- *‘There are* fewer blue cars than red cars.’





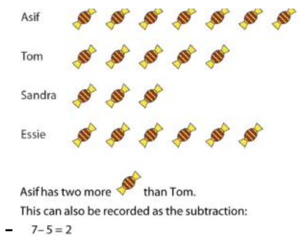
As well as presenting difference using the bar model, expose difference on a number line. Also allow opportunities for children to show difference between two numbers using these representations y themselves.

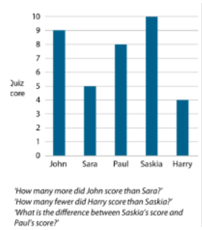
Number line:



Link difference to the subtraction calculation and encourage children to describe what the equation is showing, for example the difference between eight and three can be written as 8 – 3 =5. Unsure children are exposed to the concept that consecutive whole number have a difference of one, consecutive odd/even numbers have a difference of two.

Apply the structure of difference to compare data





Counting in tens

Teaching children that counting in groups of ten is an easier and more efficient way of counting larger amounts of items.
Teaching the concept of unitizing 1 ten

Groups (of 10)	One (s)
2	6

Read and write: 10 ones is 1 ten. 2 tens and 6 ones makes twenty-six.

Write 2 on the left. It means 2 tens.

Write 6 on the right. It means 6 ones.

Give opportunities for children to tie up sticks/straws of 10 and work it out themselves using the recording and the place value chart.

2 tens and 3 ones is 23.
20 + 3 = 23

Ten(s)	One(s)
2	3

Counting to 100: adding and subtracting multiples of 10

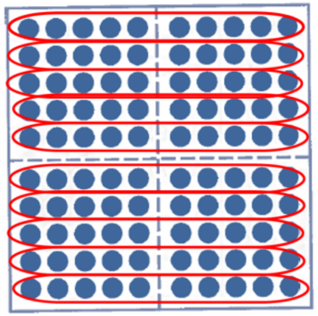
Packing kiwi fruits

Boxes Hundreds H	Trays Tens T	Ones O

Hundreds H	Tens T	Ones O
1	0	0

10 tens makes 1 hundred.
100 is written with 1 in the hundreds place and 0s in the tens and ones places. 1 in the hundreds place stands for 1 hundred.

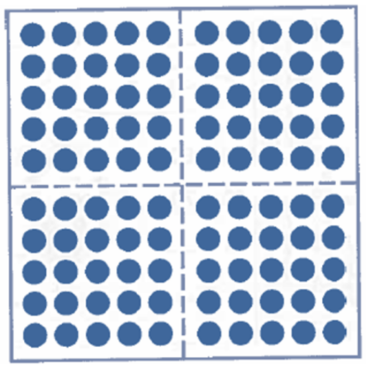
The 100 square. What do you notice?



1 ten	10	ten
2 tens	20	twenty
_tens	_	thirty
_tens	_	forty
_tens	_	fifty
_tens	_	sixty
_tens	_	seventy
_tens	_	eighty
_tens	_	ninety
_tens	_	one hundred

10 tens makes 100

Use the 100 square to help us calculate

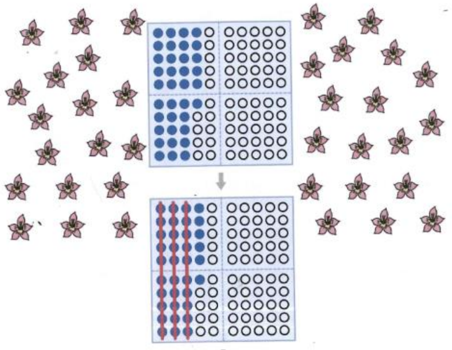


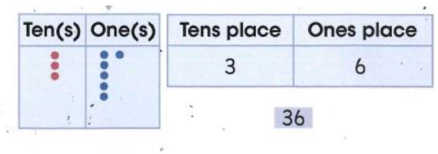
10	One 10
10 + 10 =	Two 10s
20 + 10 =	Three 10s
30 + 10 =	Four 10s
= 50	Five 10s
50 + 10 =	Six 10s
50 + 20 =	Seven 10s
50 + 30 =	Eight 10s
50 + 40 =	Nine 10s
50 + 50 =	Ten 10s

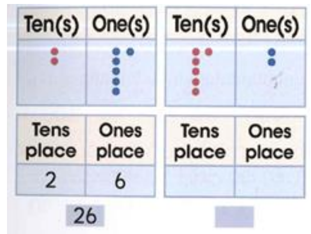
Using this 100 square, ask children what they notice. Focus in the tens but some children will see it as 4 parts, which equals 25. At 5 tens – ask children what they notice linked to the 100 square. This 100 square can be used to support children to calculate, for example 40 + 50 = 4 tens add 5 tens = 9 tens, which is ninety, and 70 – 20 = 7 tens subtract 2 tens = 5 tens, which is fifty.

Representing numbers to 100.

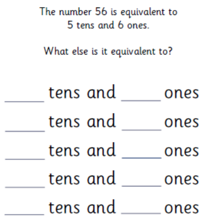
Children must understand the concept that 1 ten makes 10 ones. They will develop their understanding of place value with numbers up to 100.

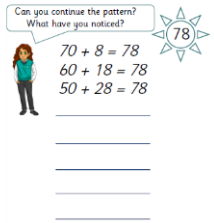






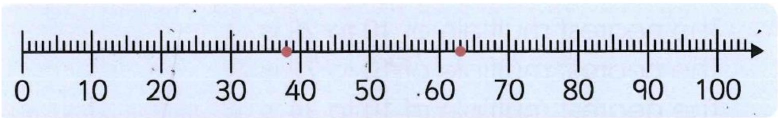
Children must be able to partition two-digit numbers into different combinations of tens and ones (verbal, pictures or apparatus). Expose patterns and efficient strategies.





Comparing numbers up to 100

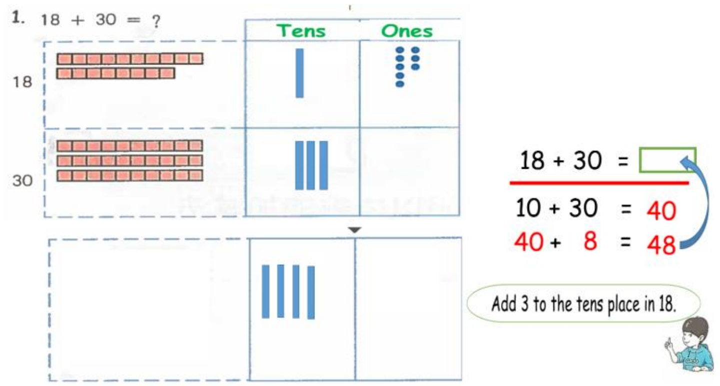
Children need to be able to use < > = symbols to compare numbers. Eg which is greater: 63 or 38? Counting in ones from 0:1,2,3,4….the number after is greater than the number before. Mark these numbers on a number line. Children have already learned that the number on the right is greater than the number on the left.



Answer: 63 is more than 38.

Children need to compare two, two-digit numbers. The number with the greater digit in the tens place is the greater number. If both digits in the tens place are the same, then the number with the greater digit in the ones place is greater.

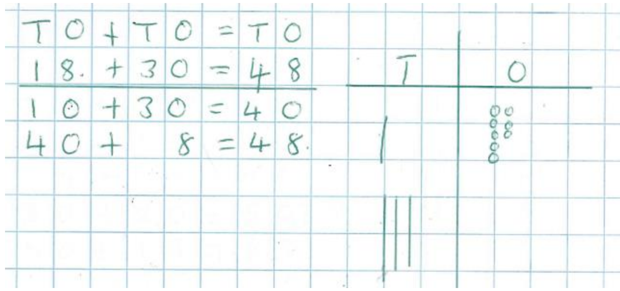
Adding and subtracting tens to or from a two-digit number



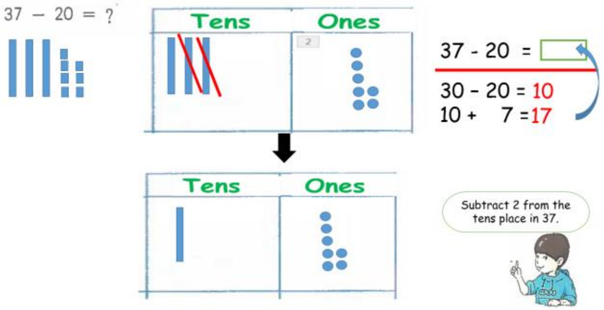
Challenge 1: add these numbers. Look for patterns.

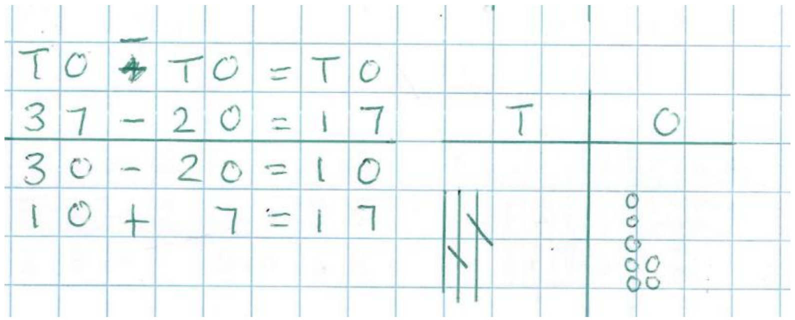
49 + 10 =	36 + 10 =	10 + 21 =
49 + 20 =	36 + 20 =	20 + 21 =
49 + 30 =	36 + 30 =	70 + 21=

Challenge 2:
10 + 67 =
67 + 20 =
30 + 67 =

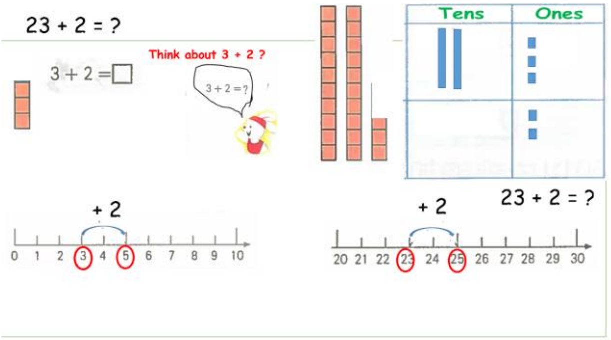


Talk about the ones and pay attention to the fact that there are no ones in 30.
Model how to record written method on w/b too alongside the dienes jottings to support children who need it.
Make sure the one are always representing the ten frame model to support calculating.
Add number which look for patterns first and then challenge. For example:

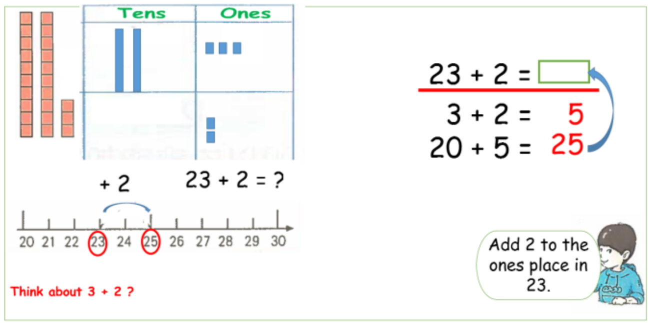


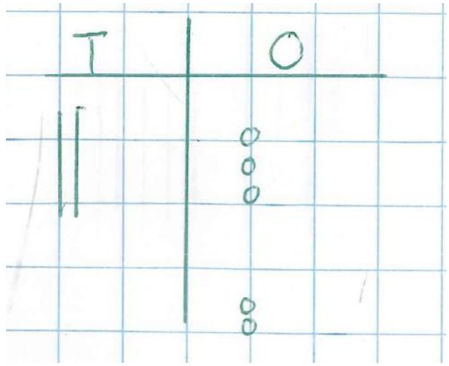


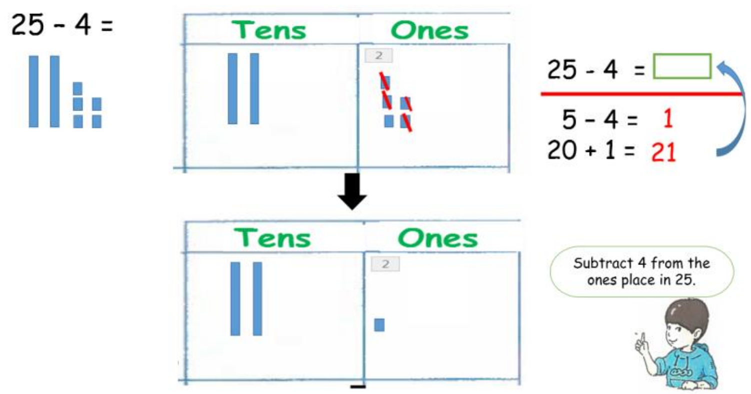
Adding and subtracting a one-digit number to or from a two-digit number

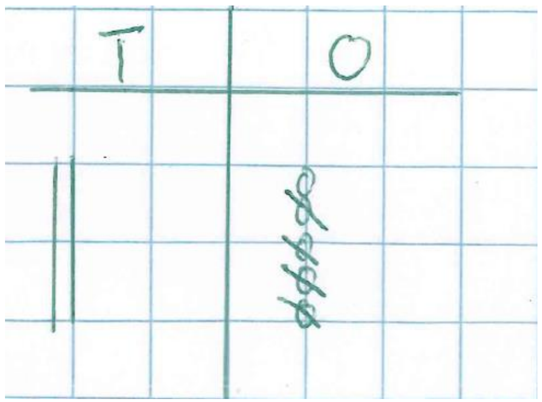


23 + 2 = ? Let's look at how to record this using a method and jottings.





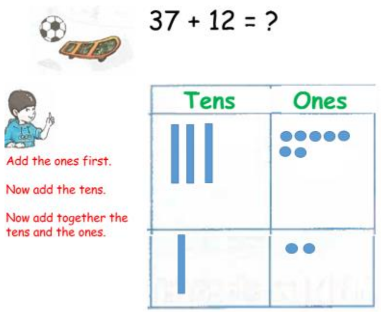




Addition 2 digit numbers:

Model how to add together 2 digit numbers using dienes and images

Add together the ones first then add the ten. Use the dienes to model



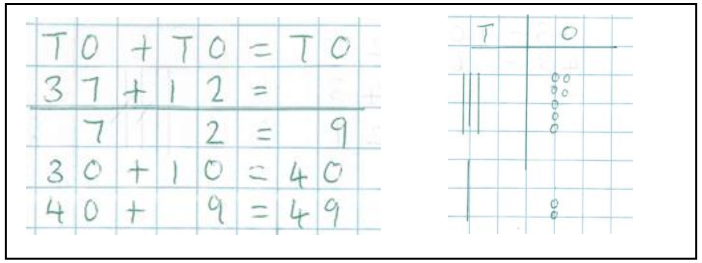
$37 + 12 = \boxed{}$

$7 + 2 = 9$

$30 + 10 = 40$

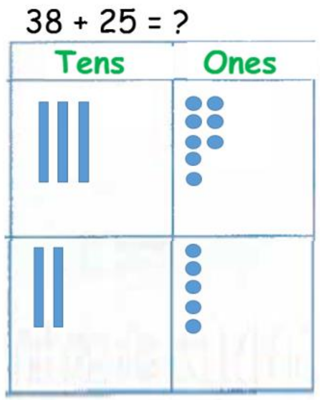
$40 + 9 = 49$

Make sure ten frames are used to support understanding.



Children should use jottings of dienes to add to together the 2 digit numbers and record as above

Addition 2 digit numbers when re-grouping is required



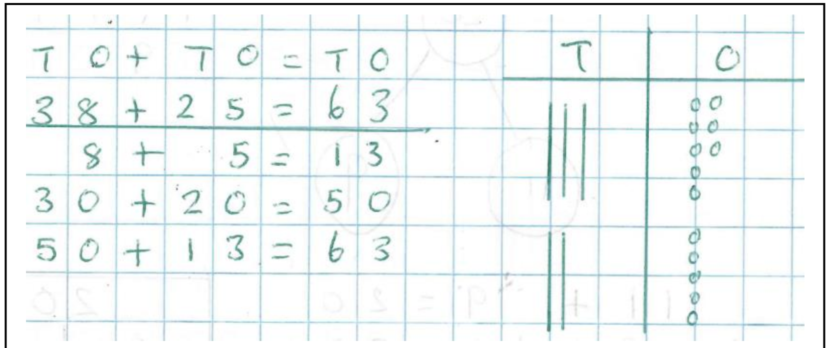
$38 + 25 = \boxed{63}$

$30 + 20 = 50$

$8 + 5 = 13$

$50 + 13 = 63$

I would calculate using this method.



Children to use jottings of the dienes to show the addition and the exchange. The record addition.

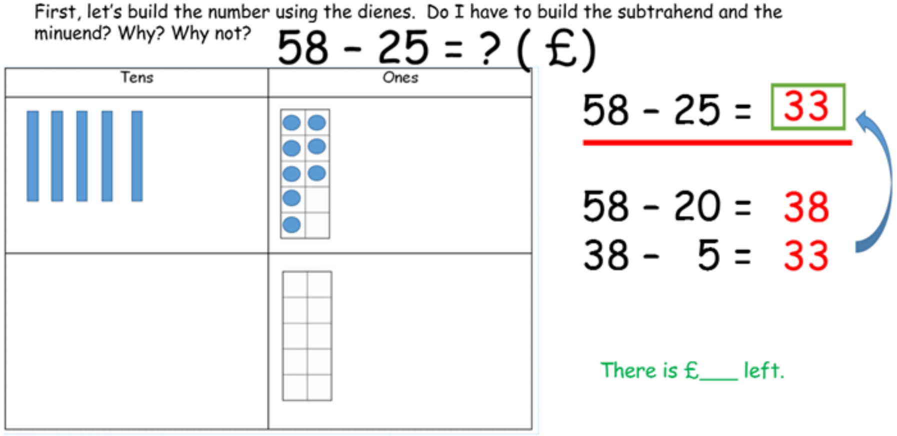
Subtracting a two-digit number from a two-digit number

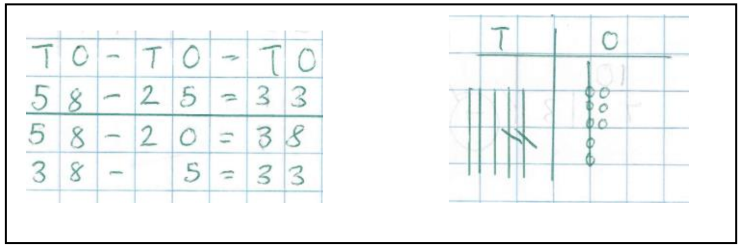
Teach partitioning the second number, e.g

85 – 34 (underline the second number)

85 – 30 = 55

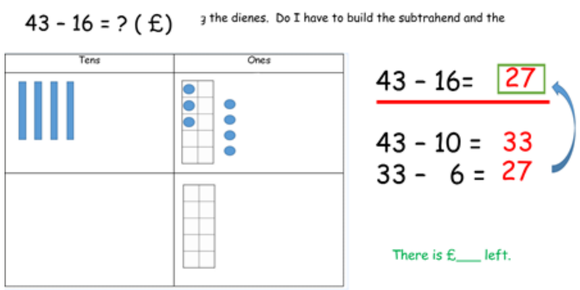
55 – 4 = 51





Children must use jottings to support their understanding until they are ready to move away from using the jottings alongside.

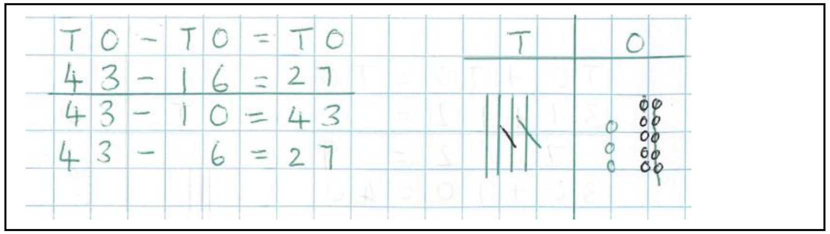
Subtracting a two-digit number from a two-digit number crossing the tens boundary



Step 1. Take away the tens.

Step 2. Exchange 1 ten for 10 ones.

Step 3. Take away the tens.



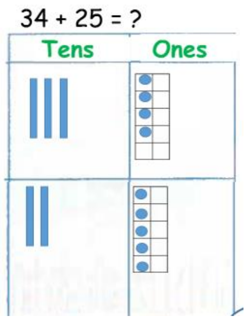
It is essential that you model using dienes to build conceptual understanding. The children must use jottings of dienes and ones to show their understanding and not just the number sentence. They can use jottings during test papers.

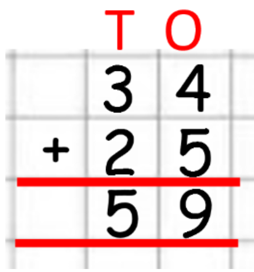
Summer term – after SATs and moderation children should be taught column addition

Introduced to column method/

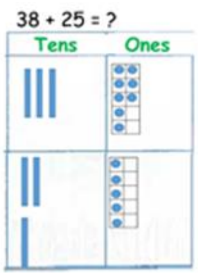
Review learning of column addition **without** crossing the boundary first. Ensure you refer to the value of the digitise g. 4 ones add 3 ones is equal to 7 ones. 2 tens add five tens is equal to 7 tens.

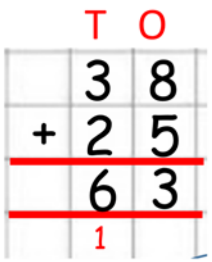
Children should use jottings alongside to support their understanding and then move away as soon as they are secure. Make sure the columns are labelled and images are used alongside the written method.





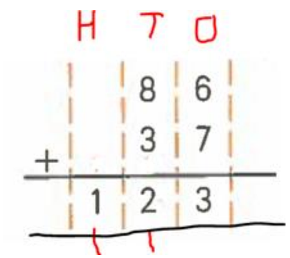
During the same lesson move onto crossing boundaries as for the example below.





Steps for addition:

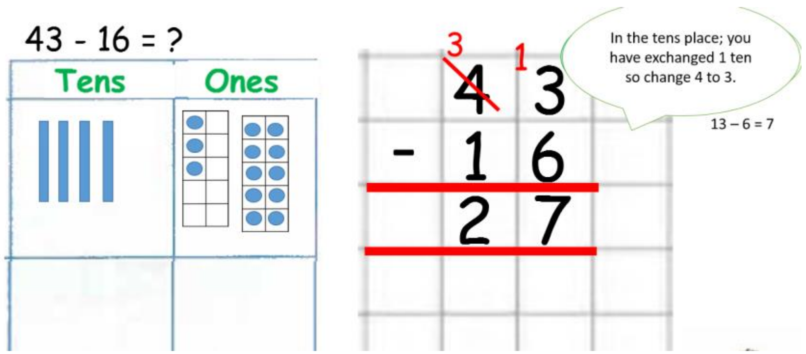
1. Keep the digits aligned in the correct columns.
2. Calculate from the ones place.
3. Exchange 10 ones for 1 ten and carry the digit.
4. Add the tens

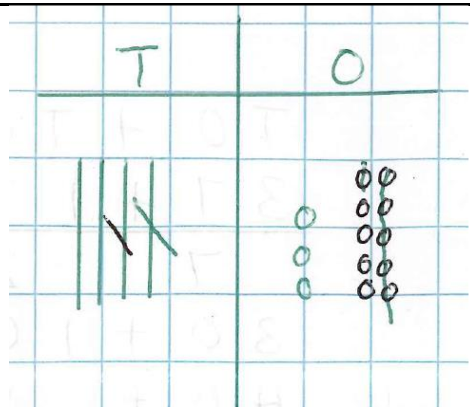


Give children opportunity to cross the hundreds boundary. E.g the example given.

Introduce column subtraction

Introduce the concept with crossing the tens boundary to begin with.





Known facts	Recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100.	
Essential Knowledge	10 more	Number bonds:20,12 and 13
	Add 1 digit to 2 digit by bridging	Number bonds: 14 and 15
	Partition second number and add tens then ones.	Number bonds: 16 and 17
	Add 10 and multiples of 10.	Number bonds: 18 and 19
	Doubles up to 20 and multiples of 5.	Partition and recombine.
	Add near multiples of 10.	